Hall Ticket Number:

Code No. : 1102S

VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD
B.E. I Year I - Semester (Supplementary) Examinations, July/Aug - 2015

Mathematics - I

Time: **3** hours Max. Marks: **70**

*Note: Answer **ALL** questions in **Part-A** and any **FIVE** questions from **Part-B***

Part-A (10 X 2=20 Marks)

1. For what values of k, the equations $x+y+z=1,\ 2x+y+4z=k, 4x+y+10z=k^2$ have a solution.

2. Define (i) Positive definite (ii) Negative definite (iii) Positive Semi-definite (iv) Negative Semi-definite quadratic forms.

3. Test the convergence of the series $\sum \frac{\sqrt{n+1}-\sqrt{n}}{n^p}$

4. Test the convergence of the series $\sum n e^{-n^2}$

5. Find the radius of curvature at the origin of the curve $y=x^4-4x^3-18x^2$

6. Find the envelope of the family of the curves $y=mx+\sqrt{a^2m^2+b^2}$, m is a parameter.

7. If $x^y+y^x=c$ find $\frac{dy}{dx}$

8. If the kinetic energy $T=\frac{mv^2}{2}$. Find approximately the change in T as m changes from 49 to 49.5 and v changes from 1600 to 1590.

9. Evaluate $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)dx\,dy$

10. Evaluate $\int_1^c\int_1^{\log y}\int_1^{e^x}\log z\,dz\,dx\,dy$

Cont..2..

Part-B (Marks: 5x10=50)
(All bits carry equal marks)

11. (a) If $A=\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$, Evaluate A^{-1}, by using Cayley Hamiton theorem.

(b) Show that the sum of the eigen values of a matrix A is the sum of the elements of the principal diagonal and product of eigen values of a matrix A is equal to its determinant.

12. (a) Discuss the convergence of the series

$$\frac{x}{1}+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{x^5}{5}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{x^7}{7}+\ldots\ldots\ldots\ldots(x>0)$$

(b) Prove that the series $\frac{\sin x}{1^3}-\frac{\sin 2x}{2^3}+\frac{\sin 3x}{3^3}$.................converges absolutely

13. (a) Show that the evolute of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ is $(x+y)^{2/3}+(x-y)^{2/3}=2a^{2/3}$

(b) Trace the curve $x=a(\theta+\sin\theta)$, $y=a(1-\cos\theta)$

14. (a) If $u=f(r)$ and $r=(x^2+y^2+z^2)^{\frac{1}{2}}$. Prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f''(r)+\frac{2}{r}f'(r)$

(b) If $u^3+v^3+w^3=x+y+z$, $u^2+v^2+w^2=x^2+y^2+z^2$, $u+v+w=x^3+y^3+z^3$

then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$

15. (a) Change the order of integration and evaluate $\int_0^1\int_x^{\sqrt{2-x^2}}\frac{x}{\sqrt{x^2+y^2}}dydx$

(b) Evaluate $\iint_R xy(x+y)dxdy$ over the region R bounded by the curve $y=x^2$ and $y=x$

16. (a) Reduce the quadratic form $x^2-4y^2+6z^2+2xy-4xz+2w^2-6zw$ into sum of squares form. Hence find rank, index and signature of the quadratic form.

(b) Expand $(1+x)^x$ in powers of x.

17. (a) Find the maximum and minimum values of $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$

(b) Expand $e^y \sin x$ by using Maclurin's development series up to 3rd degree.

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